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Vector supersymmetry in topological field theories.

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Abstract: We present a simple derivation of vector supersymmetry transformations for topological field theories of Schwarz- and Witten-type. Our method is similar to the derivation of BRST-transformations from the so-called horizontality conditions or russian formulae. We show that this procedure reproduces in a concise way the known vector supersymmetry transformations of various topological models (in particular of general relativity viewed as a topological-like model) and we use it to obtain some new transformations of this type for $4d$ topological YM-theories in different gauges.

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Contents

1. Introduction

It is well known that there exist two classes of topological quantum field theories (TQFT's) [1]. The so-called *Witten-type models*, e.g. topological Yang-Mills (YM) theory on a four-dimensional space-time manifold [2]-[6] and its higher-dimensional generalizations [7] and the *Schwarz-type models*, e.g. the Chern-Simons theory in any odd space-time dimension and the BF model in arbitrary dimensions. These models are not only invariant under BRST-transformations, but also (at least in certain gauges) under the so-called *vector supersymmetry* (VSUSY). The anticommutator of this transformation with the BRST-operator yields space-time translations (either off-shell or on-shell) and thus generates a superalgebra of Wess-Zumino type [8, 9, 10]. The VSUSY plays a central role for proving the perturbative finiteness of Schwarz-type models (e.g. see [9]) and it is most helpful for discussing the algebraic renormalization of Witten-type models [6].

In reference [11], a general procedure for obtaining the explicit form of VSUSY-transformations was presented for the three-dimensional BF-system (see also [9] for the three-dimensional Chern-Simons theory). However, this derivation becomes rather involved for more complex models. Another approach to VSUSY consists of writing the most general Ansatz for these transformations which is compatible with dimensional and covariance constraints and subsequently eliminating terms by imposing the aforementioned anticommutation relations and the invariance of the action. In practice, this also turns out to be a laborious undertaking.

The aim of the present work is to give a short derivation of the two different forms of VSUSY's appearing in topological models. Our results also allow us to cast these transformations into a compact form. The proposed procedure closely parallels the derivation of BRST-transformations in field theories with local symmetries from the so-called horizontality conditions or russian formulae [12, 13, 14]. The latter enclose all field variations in a single (or a few) simple equation(s). Though the horizontality conditions admit a clear geometric interpretation in the case of ordinary YM-theories [14], they seem to be a bit mysterious for more general field theoretic models. Therefore, we will first provide some insight into their working mechanism. In particular, we will emphasize that they not only encode all information concerning the explicit form of BRST-transformations, but also about their nilpotency.

Our paper is organized as follows. Section 2 is devoted to reviewing the horizontality conditions for some of the prototype models of TQFT mentioned above. In section 3, we discuss the VSUSY before presenting our general derivation of VSUSY-transformations in section 4. Some comments are gathered in a concluding section. We note that all of our considerations concern the classical theory (tree-level).

2. BRST-symmetry in topological field theories

As prototype models, we consider topological YM-theory on a riemannian 4-manifold $\mathcal{M}_4$, Chern-Simons theory on $\mathbb{R}^3$ and the BF model on a riemannian manifold $\mathcal{M}_{p+2}$ of dimension $p+2$. All fields in these models are given by differential forms with values in a Lie algebra. The local symmetries will be described in the BRST-framework, i.e. infinitesimal symmetry parameters are turned into ghost fields and symmetry transformations are collected in a BRST-transformation. Thus, all fields are characterized by their form degree and ghost-number which we specify by means of lower and upper indices for the fields. All models under consideration involve a YM-connection one-form[1] $A = A^a_\mu T_a dx^\mu$. The associated field strength is defined by

$$F = dA + \frac{1}{2}[A, A]\,, \tag{2.1}$$

where $[.,.]$ denote graded brackets. From the nilpotency of the exterior derivative d, it follows that F satisfies the Bianchi identity: $DF \equiv dF + [A, F] = 0$.

[1] Here, the T_a represent a basis of the Lie algebra and they are assumed to satisfy $[T_a, T_b] = \mathrm{i} f^c_{ab} T_c$ and $\mathrm{tr}(T_a T_b) = \delta_{ab}$.

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It is convenient to combine the gauge field and the associated ghost c as well as the exterior derivative and the BRST-operator s by introducing the expressions

$$\begin{aligned} \tilde{A} &= A + c \\ \tilde{d} &= d + s \, . \end{aligned} \tag{2.2}$$

Accordingly, one defines

$$\begin{aligned} \tilde{F} &= \tilde{d}\tilde{A} + \frac{1}{2}[\tilde{A}, \tilde{A}] \\ \tilde{D} &= \tilde{d} + [\tilde{A}, \cdot] \, . \end{aligned} \tag{2.3}$$

By definition, the s-operator anticommutes with d, and d is nilpotent, henceforth

$$\tilde{d}^2 = s^2 \qquad \text{on all fields.} \tag{2.4}$$

By expanding $\tilde{F}$ with respect to the ghost-number, we find that it has an expression of the form

$$\tilde{F} = F_2^0 + F_1^1 + F_0^2 \, , \tag{2.5}$$

where

$$\begin{aligned} F_2^0 &\equiv F \\ F_1^1 &\equiv sA + Dc \\ F_0^2 &\equiv sc + \frac{1}{2}[c, c] \, . \end{aligned} \tag{2.6}$$

From equations (2.2)-(2.4), it follows that

$$\tilde{D}\tilde{F} = \tilde{d}^2\tilde{A} = s^2 A + s^2 c \tag{2.7}$$

and similarly equations (2.3)-(2.5) imply

$$\tilde{D}(\tilde{D}\tilde{F}) = \tilde{d}^2\tilde{F} = s^2 F + s^2 F_1^1 + s^2 F_0^2 \, . \tag{2.8}$$

So far, we have simply derived some equations involving s-variations of A and c without specifying the latter. According to equations (2.5) and (2.6), these can be determined by imposing a *horizontality condition*, i.e. by prescribing F_1^1 and F_0^2 in equation (2.5), F_2^0 being necessarily equal to $F \equiv dA + \frac{1}{2}[A, A]$. Equations (2.7) and (2.8) then allow us to discuss the nilpotency of the resulting BRST-transformations and thereby to check the consistency of the imposed horizontality condition.

2.1 Witten-type models

Topological Yang-Mills theory. The classical action reads

$$\Sigma_{\rm inv}^{W} = \int_{\mathcal{M}_4} {\rm tr}\,(FF)\,, \tag{2.9}$$

where the wedge product symbol has been omitted.

Due to the *shift-* (or topological Q-) *symmetry* $\delta A = \psi_1^1$ which is present in this type of model [2]-[6], the connection A is associated with ghost fields ψ_1^1 and $\varphi_0^2 \equiv \varphi^2$. Henceforth, one imposes the horizontality condition [3]

$$\tilde{F} = F + \psi_1^1 + \varphi^2\,. \tag{2.10}$$

Substitution of equations (2.5),(2.6) into this relation yields the BRST-transformations

$$\begin{aligned} sA &= \psi_1^1 - Dc \\ sc &= \varphi^2 - \frac{1}{2}[c,c]\,. \end{aligned} \tag{2.11}$$

Since sA involves an inhomogeneous term ψ_1^1, the requirement $s^2A = 0$ determines $s\psi_1^1$ in terms of the other fields and analogously the condition $s^2c = 0$ determines $s\varphi^2$. In order to obtain the explicit form of these variations, as well as the one of F, we note that substitution of $s^2A = 0 = s^2c$ in equation (2.7) yields the generalized Bianchi identity $\tilde{D}\tilde{F} = 0$. By expanding the latter with respect to the ghost-number, one readily obtains

$$\begin{aligned} sF &= -D\psi_1^1 - [c,F] \\ s\psi_1^1 &= -D\varphi^2 - [c,\psi_1^1] \\ s\varphi^2 &= -[c,\varphi^2]\,. \end{aligned} \tag{2.12}$$

Finally, substitution of $\tilde{D}\tilde{F} = 0$ into equation (2.8) allows us to conclude that $s^2(F,\psi_1^1,\varphi^2) = 0$. Last, but not least, it can be verified explicitly that the transformation (2.12) of F leaves the classical action (2.9) invariant.

2.2 Schwarz-type models

Chern-Simons theory. The classical action of this model is given by

$$\Sigma_{\rm inv}^{CS} = \frac{1}{2}\int_{\mathbb{R}^3} {\rm tr}\left(AdA + \frac{1}{3}A[A,A]\right)\,. \tag{2.13}$$

Due to the absence of the shift-symmetry in this model, one imposes the horizontality condition

$$\tilde{F} = F\,, \tag{2.14}$$

which implies

$$\begin{aligned} sA &= -Dc \\ sc &= -\frac{1}{2}[c,c]\,. \end{aligned} \tag{2.15}$$

From equations (2.11) and (2.12), we see that the truncation (2.14) of (2.5) is consistent and that it leads to nilpotent s-variations.

BF model. Apart from a YM 1-form A and its associated ghost c, this model involves a p-form $B \equiv B_p^0$ transforming with the adjoint representation of the gauge group. Its field strength $H \equiv DB$ automatically satisfies the second Bianchi identity $DH = [F, B]$. The model is characterized by the classical action

$$\Sigma_{\text{inv}}^{BF} = \int_{\mathcal{M}_{p+2}} \text{tr}(BF)\,, \tag{2.16}$$

which is not only invariant under ordinary gauge transformations, but also under the reducible local symmetry $\delta B = DB_{p-1}^1$. Henceforth, the field B is associated with a series of ghosts $B_{p-1}^1, B_{p-2}^2, \ldots, B_0^p$ which can be combined in a generalized field $\tilde{B}$, by analogy to the definition of $\tilde{A}$ in the YM-sector:

$$\tilde{B} = B + B_{p-1}^1 + \cdots + B_1^{p-1} + B_0^p\,. \tag{2.17}$$

Then, the generalized field strength

$$\tilde{H} \equiv \tilde{D}\tilde{B}$$

admits the expansion

$$\tilde{H} = H_{p+1}^0 + H_p^1 + \cdots + H_0^{p+1}\,, \tag{2.18}$$

where

$$\begin{aligned} H_{p+1}^0 &= DB_p^0 \\ H_p^1 &= DB_{p-1}^1 + sB_p^0 + [c, B_p^0] \\ &\vdots \\ H_0^{p+1} &= sB_0^p + [c, B_0^p]\,. \end{aligned} \tag{2.19}$$

Due to the absence of shift-symmetries, we proceed by analogy with the Chern-Simons model and truncate the expansions of the field strengths, i.e. we impose the horizontality conditions

$$\begin{aligned} \tilde{F} &= F \\ \tilde{H} &= H\,. \end{aligned} \tag{2.20}$$

From equations (2.19), we then obtain the BRST-transformations

$$\begin{aligned} sB^q_{p-q} &= -DB^{q+1}_{p-q-1} - \left[c, B^q_{p-q}\right] \qquad \text{for } q = 0, 1, \ldots, p-1 \\ sB^p_0 &= -\left[c, B^p_0\right], \end{aligned} \tag{2.21}$$

the transformations of the connection A and ghost field c being given by equations (2.15).

In order to check the nilpotency of the s-variations (2.21), we note that

$$s^2 B + s^2 B^1_{p-1} + \cdots + s^2 B^p_0 = \tilde{d}^2 \tilde{B} = \tilde{D}\tilde{H} - [\tilde{F}, \tilde{B}]\,,$$

which implies (by matching the ghost-numbers on the left and right hand sides)

$$\begin{aligned} s^2 B^q_{p-q} &= -\left[F, B^{q+2}_{p-q-2}\right] \qquad \text{for } q = 0, 1, \ldots, p-2 \\ s^2 B^q_{p-q} &= 0 \qquad \text{for } q = p-1, p\,. \end{aligned} \tag{2.22}$$

Here, the right hand side vanishes, if we use the equation of motion $F = 0$ following from the classical action (2.16). Thus, the s-variations (2.21) are only nilpotent on-shell [15, 9]. The origin of this result can be drawn back to the fact that the used horizontality conditions (which were motivated by the absence of shift-symmetries) enforced a truncation of the ghost-expansion of $\tilde{H}$, which is not consistent in the sense that it leads to an on-shell algebra. This kind of phenomenon is familiar from supersymmetric field theories where the elimination of auxiliary fields from a superfield expansion leads to a supersymmetry algebra which only closes on-shell. In the BRST-framework, the on-shell closure of the symmetry algebra is reflected by the fact that the s-variations of the ghost fields are only nilpotent on-shell. From our discussion, it followed that this information can be directly extracted from the horizontality conditions.

3. Vector supersymmetry

In flat space-time, infinitesimal VSUSY-transformations are parametrized by a constant vector field. On curved space-time manifolds, one has to consider a covariantly constant vector field [16]. Although this can be done at the expense of technical complications, we will limit our discussion of VSUSY to flat space-time for the sake of simplicity.

The total action $\Sigma = \Sigma_{\text{inv}} + \Sigma_{gf}$ of a topological model not only involves classical and ghost fields, but also anti-ghost and Lagrange multiplier fields.[2] Let us denote all these fields collectively by $(\Phi_i)_{i=1,2,\ldots}$. Their infinitesimal VSUSY-variations $\delta_\tau \Phi_i \equiv$

[2]Here, "classical" fields are not opposed to quantum fields, but simply refer to the fields appearing in the classical action.

$\tau^\mu \delta_\mu \Phi_i$ are parametrized by a constant, s-invariant vector field $\tau = \tau^\mu \partial_\mu$ of ghost-number zero. The operator δ_τ acts as an antiderivation which lowers the ghost-number by one unit and which anticommutes with d.

The existence and explicit form of VSUSY-transformations for a topological model described by the action Σ depends, in general, in a sensitive way on the choice of the gauge-fixing condition. In order to study the existence of this symmetry and to determine the explicit form of VSUSY-transformations, one can apply a general procedure presented in reference [11] (see also [9]). This method is based on the facts that the gauge-fixing term is BRST-exact and that it is metric dependent (while the classical action is metric independent): thus, the energy-momentum tensor is a BRST-exact expression,

$$T_{\mu\nu} = s\Lambda_{\mu\nu} \,, \tag{3.1}$$

which is a typical feature of topological models [2, 1]. After determining $\Lambda_{\mu\nu}$, one expresses $\partial^\mu \Lambda_{\mu\nu}$ in terms of the functional derivatives $\delta\Sigma/\delta\Phi_i$, thereby producing *contact terms*, i.e. expressions which vanish when the equations of motion are used. If

$$\partial^\mu \Lambda_{\mu\nu} = \text{contact terms} + \partial^\mu \varepsilon_{\mu\nu} \,, \qquad \text{where} \quad s(\partial^\mu \varepsilon_{\mu\nu}) = 0 \,, \tag{3.2}$$

then the quantities

$$\begin{aligned} \hat{\Lambda}_{\mu\nu} &= \Lambda_{\mu\nu} - \varepsilon_{\mu\nu} \\ \hat{T}_{\mu\nu} &= T_{\mu\nu} - s\varepsilon_{\mu\nu} \end{aligned} \tag{3.3}$$

are conserved up to equations of motion. They are still related by $\hat{T}_{\mu\nu} = s\hat{\Lambda}_{\mu\nu}$ and $\hat{T}_{\mu\nu}$ can be viewed as an improvement of the energy-momentum tensor. More explicitly, one finds

$$\partial^\nu \hat{\Lambda}_{\nu\mu} = \text{tr}\left(V_{i\mu} \frac{\delta\Sigma}{\delta\Phi_i} \right) ,$$

where $V_{i\mu}$ are polynomials in the fields Φ_i and their derivatives. Integration of the last equation over the n-dimensional space-time on which the topological model is defined, yields

$$0 = \int_{\mathcal{M}_n} d^n x \, \text{tr}\left(V_{i\mu} \frac{\delta\Sigma}{\delta\Phi_i} \right) . \tag{3.4}$$

This relation expresses the invariance of Σ under the VSUSY-transformations $\delta_\mu \Phi_i := V_{i\mu}$. A nice feature of this approach is that, by construction, the obtained variations of the fields represent a symmetry of the theory. Yet, for a given TQFT, it may be quite tedious to carry out the calculations.

For all known models, the VSUSY- and BRST-operators satisfy the anticommutation relations

$$[s, \delta_\tau]\Phi_i = \mathcal{L}_\tau \Phi_i + \text{contact terms} \,. \tag{3.5}$$

Here, $\mathcal{L}_\tau = [i_\tau, d]$ represents the Lie derivative along the vector field τ and i_τ denotes the interior product with τ. Since the algebra closes on space-time translations, it describes a superalgebra of Wess-Zumino type and, for brevity, we will refer to (3.5) as the *SUSY-algebra*. More precisely, this algebra closes off-shell for Witten-type models and on-shell for Schwarz-type models. The lack of off-shell closure for the latter theories can be explained by the fact that the shift-symmetry and thereby the associated ghosts are "missing".

In the next section, we will use differential forms to present the known, as well as some new, results for our prototype models. Before doing so, we summarize some useful algebraic relations.

The graded commutation relations between the basic operators read:

$$\begin{aligned} 0 &= [s, d] = [s, i_\tau] = [s, \mathcal{L}_\tau] \\ 0 &= [\delta_\tau, d] = [\delta_\tau, i_\tau] = [\delta_\tau, \mathcal{L}_\tau] \,. \end{aligned} \tag{3.6}$$

As usual, the Hodge dual of a Lie algebra-valued differential form Ω will be denoted by $*\Omega$. On a n-dimensional space-time manifold $\mathcal{M}_n$, the star operator can be used to define a scalar product of Lie algebra-valued p-forms Ω_p^q which, in addition, have some ghost-number q:

$$\langle \Omega_p^q, \Lambda_p^r \rangle \equiv \int_{\mathcal{M}_n} \mathrm{tr}\, (\Omega_p^q \, * \Lambda_p^r) \,.$$

This product has the graded symmetry

$$\langle \Omega_p^q, \Lambda_p^r \rangle = (-1)^{(p+n)(q+r)+qr} \langle \Lambda_p^r, \Omega_p^q \rangle \,. \tag{3.7}$$

If the space-time dimension is odd, the star operation represents a mapping between forms of even and odd degree, henceforth it anticommutes with the antiderivation s.

By using the metric tensor $(g_{\mu\nu})$ of the space-time manifold, one can associate the 1-form $g(\tau) \equiv \tau^\mu g_{\mu\nu} dx^\nu$ to the vector field $\tau = \tau^\mu \partial_\mu$. (In the mathematical literature, this mapping and its inverse are known as the "musical isomorphisms", which are usually denoted by $\flat$ and $\sharp$, respectively [17].) The Hodge operator intertwines between the interior product i_τ and the exterior multiplication with $g(\tau)$:

$$\begin{aligned} * \, g(\tau)\, \Omega_p^q &= (-1)^p \, i_\tau * \Omega_p^q \\ g(\tau) * \, \Omega_p^q &= (-1)^{p+1} \, * i_\tau \Omega_p^q \,. \end{aligned} \tag{3.8}$$

3.1 Witten-type models

In the literature, two different types of gauge-fixings have been considered for topological YM-theory.

- The first choice consists of a linear gauge condition for both the shift-symmetry and the ordinary gauge symmetry [6]:

$$\Sigma_1^W = \int_{\mathcal{M}_4} \mathrm{tr}\,(FF) + s \int_{\mathcal{M}_4} \mathrm{tr}\left\{\chi_2^{-1} F^+ + \bar{\phi}^{-2} d * \psi_1^1 + \bar{c} d * A\right\}. \tag{3.9}$$

Here, the fields $F^+ \equiv \frac{1}{2}(F + *F)$, χ_2^{-1} and $s\chi_2^{-1} \equiv B_2$ are self-dual and the BRST-variations are defined by (2.11),(2.12) and

$$\begin{aligned} s\chi_2^{-1} &= B_2\,, & sB_2 &= 0\,,\\ s\bar{\phi}^{-2} &= \eta^{-1}\,, & s\eta^{-1} &= 0\,,\\ s\bar{c} &= b & sb &= 0\,. \end{aligned} \tag{3.10}$$

The action (3.9) is also invariant under the following VSUSY-variations [6]:

$$\begin{aligned} \delta_\tau A &= 0\,, & \delta_\tau \psi_1^1 &= \mathcal{L}_\tau A\\ \delta_\tau c &= 0\,, & \delta_\tau \varphi^2 &= \mathcal{L}_\tau c \end{aligned} \tag{3.11}$$

and

$$\begin{aligned} \delta_\tau \bar{c} &= -\mathcal{L}_\tau \bar{\phi}^{-2}\,, & \delta_\tau b &= \mathcal{L}_\tau \bar{c} + \mathcal{L}_\tau \eta^{-1}\\ \delta_\tau \chi_2^{-1} &= 0\,, & \delta_\tau B_2 &= \mathcal{L}_\tau \chi_2^{-1}\,,\\ \delta_\tau \bar{\phi}^{-2} &= 0\,, & \delta_\tau \eta^{-1} &= \mathcal{L}_\tau \bar{\phi}^{-2}\,. \end{aligned} \tag{3.12}$$

- The second choice is as follows [3, 5, 4]. For the shift-symmetry, one considers a covariant gauge condition and for the ordinary gauge symmetry, one chooses either (a) a linear, (b) a covariant or (c) no gauge condition at all:

$$\Sigma_{2\alpha}^W = \int_{\mathcal{M}_4} \mathrm{tr}\,(FF) + s \int_{\mathcal{M}_4} \mathrm{tr}\left\{\chi_2^{-1} F^+ + \bar{\phi}^{-2} D * \psi_1^1\right\} + s\Psi_\alpha \qquad \text{with } \alpha \in \{a, b, c\}, \tag{3.13}$$

where

$$\Psi_a = \int_{\mathcal{M}_4} \mathrm{tr}\,\{\bar{c} d * A\}\,, \qquad \Psi_b = \int_{\mathcal{M}_4} \mathrm{tr}\,\{\bar{c} D * A\}\,, \qquad \Psi_c = 0\,. \tag{3.14}$$

The BRST-transformations are again given by (2.11), (2.12) and (3.10) (or by adding a gauge symmetry contribution $-[c, \Phi_i]$ to each variation $s\Phi_i$ in (3.10) - such a contribution does not matter for our considerations).

For the action (3.13), we have found the VSUSY-variations

$$\begin{aligned} \delta_\tau A &= 0\,, & \delta_\tau \psi_1^1 &= i_\tau F = i_\tau dA + [i_\tau A, A]\\ \delta_\tau c &= i_\tau A\,, & \delta_\tau \varphi^2 &= i_\tau \psi_1^1 \end{aligned} \tag{3.15}$$

and

$$\begin{aligned} \delta_\tau \bar{c} &= 0\,, & \delta_\tau b &= \mathcal{L}_\tau \bar{c}\\ \delta_\tau \chi_2^{-1} &= 2\left(i_\tau * D\bar{\phi}^{-2}\right)^+\,, & \delta_\tau B_2 &= \mathcal{L}_\tau \chi_2^{-1} + 2\left(i_\tau * ([\psi_1^1 - Dc, \bar{\phi}^{-2}] - D\eta^{-1})\right)^+\\ \delta_\tau \bar{\phi}^{-2} &= 0\,, & \delta_\tau \eta^{-1} &= \mathcal{L}_\tau \bar{\phi}^{-2}\,. \end{aligned} \tag{3.16}$$

By contrast to the transformations (3.11), and (3.12), these variations are *not* linear in the basic fields. Since A and $\bar{c}$ do not transform under δ_τ, the gauge-fixing term $s\Psi_\alpha$ for ordinary gauge symmetry is, taken by itself, invariant under VSUSY. In section 4.1 below, we will explain how we determined the given VSUSY-transformations and why they represent a symmetry of the action.

Both sets of VSUSY-transformations have several features in common, which can thereby be considered as characteristic for topological models of Witten-type. First, both of them fulfill the SUSY-algebra (3.5) off-shell. Second, both of them leave the classical field A inert and therefore they do not act on the classical action. Thus, they only represent a non-trivial symmetry of the gauge-fixing part of the action. This should explain the fact that the VSUSY is not restrictive enough for topological YM-theory to make the model perturbatively finite, though its existence considerably improves the algebraic renormalization procedure, leading to an anomaly-free quantized theory [6]. Finally, we remark that the classical and ghost fields, i.e. the fields which belong to the geometric part of the BRST-algebra, transform among themselves under VSUSY: none of the anti-ghosts or Lagrange multipliers involved in the gauge-fixing action appears in the variations (3.11) and (3.15).

3.2 Schwarz-type models

Chern-Simons theory. In the Landau gauge, the total Chern-Simons action is given by [8, 9]

$$\begin{aligned}\Sigma^{CS} &= \Sigma^{CS}_{\text{inv}} + \Sigma^{CS}_{gf} \\ &= \frac{1}{2}\int_{\mathbb{R}^3} \operatorname{tr}\left(AdA + \frac{2}{3}A^3\right) + \int_{\mathbb{R}^3} \operatorname{tr}\left\{bd*A + \bar{c}d*Dc\right\},\end{aligned} \tag{3.17}$$

where $\bar{c}$ and b are, respectively, the anti-ghost and the corresponding Lagrange multiplier, both forming a BRST-doublet: $s\bar{c} = b$, $sb = 0$.

Substitution of the functional derivatives of Σ^{CS} with respect to A, b and c, e.g.

$$\frac{\delta\Sigma^{CS}}{\delta A} = F - *db - [c, *d\bar{c}]\,, \tag{3.18}$$

into expression (3.4), leads to the (linear) VSUSY-variations [8, 9]

$$\begin{aligned}\delta_\tau A &= -i_\tau *d\bar{c}\,, \qquad & \delta_\tau\bar{c} &= 0 \\ \delta_\tau c &= i_\tau A\,, & \delta_\tau b &= \mathcal{L}_\tau\bar{c}\,.\end{aligned} \tag{3.19}$$

The SUSY-algebra (3.5) now closes off-shell for $c, \bar{c}$ and b, but not for the classical field A:

$$[s, \delta_\tau]A = \mathcal{L}_\tau A - i_\tau\frac{\delta\Sigma^{CS}}{\delta A}\,. \tag{3.20}$$

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From these results (and similar results for the BF model discussed below), we conclude that the VSUSY-transformations in Schwarz-type models differ substantially from those in Witten-type models: the classical fields are not invariant, but transform into anti-ghost fields, and the SUSY-algebra does not close off-shell for the classical fields. The fact that the transformations mix the classical and gauge-fixing parts of the total action renders the VSUSY highly non-trivial and constraining for the quantum theory: it is at the origin of the perturbative finiteness of these models.

We note that a (linear) VSUSY is also present if the axial gauge is chosen [18, 10], but it does not exist for a covariant gauge

$$\frac{\delta\Sigma}{\delta b} = d *A + \alpha b \qquad (\alpha \in \mathbb{R}^*) \, .$$

The fact that the presence of VSUSY implies a certain class of gauges is a feature that is reminiscent of the anti-BRST symmetry, whose presence has similar consequences (if considered in addition to the usual BRST-symmetry) [13]. However, the VSUSY has a considerably richer structure which entails interesting results for the quantum theory, which is not the case for the anti-BRST symmetry.

BF model. The total BF action $\Sigma^{BF} = \Sigma^{BF}_{\text{inv}} + \Sigma^{BF}_{gf}$ involves

$$\Sigma^{BF}_{gf} = s \int_{\mathcal{M}_{p+2}} \operatorname{tr} \{ \bar{c} d *A + \bar{c}_{p-1} d *B + \cdots \} \, ,$$

where $\bar{c}$ and $\bar{c}_{p-1}$ are the anti-ghosts which fix the Landau gauge in the YM- and B-field sector, respectively, and where we only wrote out the terms which are relevant here. The derivation of VSUSY-transformations proceeds along the lines of the Chern-Simons theory, though one has to take into account the fact that the s-variations (2.21) of the BF model are only nilpotent by virtue of the classical equation of motion $F = 0$ (cf. equations (2.22)): since we are now dealing with the complete, gauge-fixed action, these s-variations have to be extended in an appropriate way so as to relate to the complete equation of motion. This can be achieved by standard methods [15, 9] and the following VSUSY-transformations can be found [9]:

$$\begin{aligned} \delta_\tau A &= -i_\tau *d\bar{c}_{p-1} \\ \delta_\tau B &= (-1)^p i_\tau *d\bar{c} \\ \delta_\tau B^k_{p-k} &= i_\tau B^{k-1}_{p-k+1} \qquad \text{for } k = 1, \dots, p \, . \end{aligned} \tag{3.21}$$

For the classical fields, the SUSY-algebra (3.5) only closes on-shell:

$$\begin{aligned} [s, \delta_\tau] \, A &= \mathcal{L}_\tau A - i_\tau \frac{\delta \Sigma^{BF}}{\delta B} \\ [s, \delta_\tau] \, B &= \mathcal{L}_\tau B - i_\tau \frac{\delta \Sigma^{BF}}{\delta A} \, . \end{aligned} \tag{3.22}$$

4. Derivation of vector supersymmetry transformations

In the sequel, we will repeatedly refer to the quantity

$$i_\tau \tilde{F} = i_\tau \left(\tilde{d}\tilde{A} + \frac{1}{2}[\tilde{A}, \tilde{A}] \right) = i_\tau \tilde{d}\tilde{A} - [\tilde{A}, i_\tau \tilde{A}] \, .$$

By virtue of $[i_\tau, \tilde{d}] = [i_\tau, d + s] = \mathcal{L}_\tau$ and $\tilde{D} = \tilde{d} + [\tilde{A}, \cdot\,]$, this expression takes the compact form

$$i_\tau \tilde{F} = \mathcal{L}_\tau \tilde{A} - \tilde{D} i_\tau \tilde{A} \, . \tag{4.1}$$

We now present an alternative approach to the derivation of VSUSY-transformations for topological models. To stress the analogy with the method of horizontality conditions for the derivation of BRST-transformations, we briefly summarize the main steps which are followed for the latter derivation in the case of Chern-Simons (or ordinary YM-) theory. One *assumes* that the BRST-operator is nilpotent on the fields A and c, i.e. that the graded algebra

$$[s, s] = 0 \tag{4.2}$$

holds for these fields. Then, one *postulates* the horizontality conditions involving the generalized field strength $\tilde{F} \equiv \tilde{d}\tilde{A} + \frac{1}{2}[\tilde{A}, \tilde{A}]$, i.e. for Chern-Simons theory, one postulates $\tilde{F} = F$. By expanding this relation with respect to the ghost-number, one immediately obtains the BRST-transformations. Their off-shell nilpotency, i.e. the consistency of the final equations with the starting point (4.2), can either be checked explicitly or by resorting to the general arguments indicated in section 2. As a last step, the invariance of a given action is to be verified. (Eventually, we can also reverse the problem and look for action functionals admitting the derived BRST-variations as symmetries.)

Let us now proceed with VSUSY. First of all, we *assume* that the SUSY-algebra (3.5) is satisfied *off-shell*. Next, we evaluate the δ_τ-variation of $\tilde{F}$:

$$\begin{aligned} \delta_\tau \tilde{F} &= \delta_\tau \tilde{d}\tilde{A} - [\tilde{A}, \delta_\tau \tilde{A}] \\ &= [\delta_\tau, \tilde{d}]\tilde{A} - \tilde{d}\delta_\tau \tilde{A} - [\tilde{A}, \delta_\tau \tilde{A}] \\ &= [\delta_\tau, \tilde{d}]\tilde{A} - \tilde{D}\delta_\tau \tilde{A} \, . \end{aligned} \tag{4.3}$$

Substitution of the assumed off-shell algebra entails

$$\delta_\tau \tilde{F} = \mathcal{L}_\tau \tilde{A} - \tilde{D}\delta_\tau \tilde{A} \, . \tag{4.4}$$

Comparison of the expressions (4.1) and (4.4) now motivates us to *postulate* either *$\emptyset$ -type symmetry conditions*,

$$\begin{aligned} \delta_\tau \tilde{A} &= i_\tau \tilde{A} \\ \delta_\tau \tilde{F} &= i_\tau \tilde{F} \, , \end{aligned} \tag{4.5}$$

or 0-*type symmetry conditions*,

$$\begin{aligned}\delta_\tau \tilde{A} &= 0\\ \delta_\tau \tilde{F} &= \mathcal{L}_\tau \tilde{A}\,.\end{aligned} \tag{4.6}$$

In both sets of equations, the second relation is a consequence of (or consistency condition for) the first one by virtue of equations (4.1), (4.4). The terminology $\emptyset$ versus 0 simply expresses the fact that $\delta_\tau \tilde{A} \neq 0$ as opposed to $\delta_\tau \tilde{A} = 0$.

For the B-field sector, we follow the same line of reasoning. From the definition $\tilde{H} = \tilde{D}\tilde{B}$, it follows that

$$i_\tau \tilde{H} = \mathcal{L}_\tau \tilde{B} - \tilde{D} i_\tau \tilde{B} + [i_\tau \tilde{A}, \tilde{B}]\,, \tag{4.7}$$

while the assumption that $\tilde{B}$ satisfies the SUSY-algebra off-shell, i.e. $[s, \delta_\tau]\tilde{B} = \mathcal{L}_\tau \tilde{B}$, leads to

$$\delta_\tau \tilde{H} = \mathcal{L}_\tau \tilde{B} - \tilde{D}\delta_\tau \tilde{B} + [\delta_\tau \tilde{A}, \tilde{B}]\,. \tag{4.8}$$

Comparison of both relations then motivates us again to *postulate* either $\emptyset$ *-type symmetry conditions*,

$$\begin{aligned}\delta_\tau \tilde{B} &= i_\tau \tilde{B}\\ \delta_\tau \tilde{H} &= i_\tau \tilde{H}\,,\end{aligned} \tag{4.9}$$

in conjunction with equations (4.5), or 0-*type symmetry conditions*,

$$\begin{aligned}\delta_\tau \tilde{B} &= 0\\ \delta_\tau \tilde{H} &= \mathcal{L}_\tau \tilde{B}\,,\end{aligned} \tag{4.10}$$

in conjunction with equations (4.6).

4.1 Witten-type models

Let us substitute $\tilde{A} = A + c$ and the horizontality condition for topological YM-theories, i.e. $\tilde{F} = F + \psi_1^1 + \varphi^2$, into the 0-type symmetry conditions (4.6). By decomposing with respect to the ghost-number, we immediately obtain the VSUSY-transformations (3.11). Similarly, from the $\emptyset$-type symmetry conditions (4.5), we reproduce the non-linear VSUSY-transformations (3.15). (Actually, this is how we found these variations!) Thus, the two representations of VSUSY defined, respectively, by the 0-type and $\emptyset$-type symmetry conditions, manifest themselves in Witten-type models, the symmetry depending on the chosen gauge-fixing conditions.

Let us now come back to the VSUSY-variations (3.16) of the anti-ghosts and Lagrange multipliers. The transformations of the anti-ghosts can be found by assuming that the off-shell SUSY-algebra $[s, \delta_\tau] = \mathcal{L}_\tau$ is valid, and by varying the gauge-fixing action $\Sigma_{gf} \equiv s\int L$:

$$\delta_\tau \Sigma_{gf} = \int \delta_\tau s L = \int \mathcal{L}_\tau L - s\int \delta_\tau L\,.$$

By choosing the δ_τ-variations of the anti-ghosts $(\bar{c}, \chi_2^{-1}, \bar{\phi}^{-2})$ in an appropriate way, the last term vanishes and thereby ensures the δ_τ-invariance of Σ_{gf}. Finally, the transformations of the Lagrange multipliers (b, B_2, η^{-1}) are also determined by imposing the VSUSY-algebra for them, e.g. from

$$\delta_\tau b = \delta_\tau(s\bar{c}) = \mathcal{L}_\tau \bar{c} - s(\delta_\tau \bar{c})$$

and the known δ_τ-variation of $\bar{c}$, one finds the one of b. Thus, it is by construction that the VSUSY-transformations (3.15), (3.16) represent a symmetry of the action (3.13). (The same arguments can be used to determine the δ_τ-variations (3.12) and to check the δ_τ-invariance of the action (3.9).)

Our procedure can be applied in a straightforward way to higher-dimensional TQFT's of Witten-type. In fact, in references [19], we use this approach to derive the VSUSY-transformations for various six-dimensional topological models involving abelian or non-abelian 2-form potentials [7].

As a further illustration, we will now apply our procedure to four-dimensional gravity written as a topological-like model by O. Piguet [20].

4.1.1 General relativity

The arena is a four-dimensional pseudo-riemannian manifold $\mathcal{M}$ and the basic variables of the theory are the vierbein 1-forms $e = (e^a)_{a=0,\ldots,3}$ and the Lorentz connection 1-form $\omega = (\omega^a{}_b)$. Their field strengths are the torsion $T = De \equiv de + \omega e$ and the curvature $R = d\omega + \frac{1}{2}[\omega, \omega]$.

The basic symmetries of the model are diffeomorphisms and local Lorentz transformations parametrized in a BRST-setting by ghosts $\xi = \xi^\mu \partial_\mu$ and Ω' (the latter taking its values in the Lie algebra of the Lorentz group).

Define the *generalized fields*

$$\begin{aligned} \tilde{\omega} &= \omega + \Omega' \qquad \text{with } \Omega' = \Omega + i_\xi \omega \\ \tilde{e} &= e + i_\xi e \\ \tilde{R} &= \tilde{d}\tilde{\omega} + \frac{1}{2}[\tilde{\omega}, \tilde{\omega}] \qquad \text{with } \tilde{d} = d + s \\ \tilde{T} &= \tilde{D}\tilde{e} \equiv \tilde{d}\tilde{e} + \tilde{\omega}\tilde{e}\,. \end{aligned} \tag{4.11}$$

Here, i_ξ denotes the interior product with respect to the vector field ξ and s is the BRST-differential. The BRST-transformations of all fields follow from the *horizontality conditions* [21]

$$\begin{aligned} \tilde{R} &= \mathrm{e}^{i_\xi} R \\ \tilde{T} &= \mathrm{e}^{i_\xi} T\,, \end{aligned} \tag{4.12}$$

where e^{i_ξ} denotes the exponential of the linear operator i_ξ. In fact, expansion of these relations with respect to the ghost-number yields the s-variations

$$\begin{aligned} se &= -\Omega e + L_\xi e\,, & s\Omega &= -\frac{1}{2}[\Omega,\Omega] + L_\xi \Omega \\ s\omega &= -D\Omega + L_\xi \omega\,, & s\xi &= \frac{1}{2}[\xi,\xi]\,. \end{aligned} \tag{4.13}$$

Here, $D\Omega \equiv d\Omega + [\omega,\Omega]$ is the covariant derivative of the Lie algebra-valued field Ω, L_ξ denotes the Lie derivative with respect to ξ and $[\xi,\xi]$ the graded Lie bracket of vector fields. The transformations (4.13) leave the classical action [20]

$$S_{\text{inv}} = \int_{\mathcal{M}} \varepsilon_{abcd} e^a e^b R^{cd} \tag{4.14}$$

invariant.

We now come to the derivation of VSUSY-transformations. Since the model does not admit any shift symmetries, the only possible form for the VSUSY-variations is given by the $\emptyset$-type symmetry conditions,

$$\begin{aligned} \delta_\tau \tilde{\omega} &= i_\tau \tilde{\omega} \\ \delta_\tau \tilde{R} &= i_\tau \tilde{R}\,, \end{aligned} \tag{4.15}$$

where $\tau = \tau^\mu \partial_\mu$ denotes the vector field parametrizing the symmetry.[3] Expansion of these relations with respect to the ghost-number yields the δ_τ-variations

$$\begin{aligned} \delta_\tau \omega &= 0 \\ \delta_\tau \Omega' &= i_\tau \omega\,. \end{aligned} \tag{4.16}$$

By substituting the expression of Ω' given in equations (4.11), we obtain

$$\begin{aligned} \delta_\tau \Omega' &= \delta_\tau \Omega + \delta_\tau i_\xi \omega \\ &= \delta_\tau \Omega + i_{\delta_\tau \xi} \omega\,, \end{aligned}$$

whence the result

$$\begin{aligned} \delta_\tau \Omega &= 0 \\ \delta_\tau \xi &= \tau\,. \end{aligned} \tag{4.17}$$

These transformations represent a symmetry of the gauge-fixed action [20].

4.2 Schwarz-type models

Before discussing the examples, we should note right away that the transformation laws that we will derive from the symmetry conditions in the present case, are to be considered with caution. In fact, our symmetry conditions are based on the assumption that the SUSY-algebra closes off-shell and this is not the case for the classical fields occurring in Schwarz-type models.

[3] Although our discussion of VSUSY was carried out in flat space-time, the calculation can be performed along the same lines in curved space and we have the same symmetry conditions.

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Chern-Simons theory. If we were to combine the horizontality conditions of the present model, i.e. $\tilde{F} = F$, with the 0-type symmetry conditions (4.6), we would obtain the inadmissible result $\mathcal{L}_\tau A = 0 = \mathcal{L}_\tau c$. Henceforth, the 0-type symmetry conditions (4.6) can only occur for Witten-type models where a shift-symmetry exists.

Thus, let us apply the $\emptyset$-type symmetry conditions (4.5): by decomposing the latter with respect to the ghost-number and by using $\tilde{F} = F$, we get

$$\delta_\tau A = 0\,, \qquad \delta_\tau c = i_\tau A \tag{4.18}$$

and

$$\delta_\tau F = 0\,, \qquad i_\tau F = 0\,. \tag{4.19}$$

From the transformations (4.18) and the s-variations of A and c, it follows that

$$\begin{aligned}
[s, \delta_\tau] A &= \mathcal{L}_\tau A - i_\tau F \\
[s, \delta_\tau] c &= \mathcal{L}_\tau c\,,
\end{aligned} \tag{4.20}$$

where $i_\tau F = 0$, if the classical equations of motion are used. Thus, we have obtained an on-shell algebra after having assumed the validity of an off-shell algebra as our starting point: this result is due to the truncation of the ghost-expansion $\tilde{F}$.

The algebra (4.20) can be interpreted as follows. If we only consider the classical action, the latter is invariant under the δ_τ-variations (4.18) which satisfy the SUSY-algebra on-shell. We will now try to promote this trivial symmetry of the classical action to a non-trivial symmetry of the total action (again allowing for an on-shell closure of the SUSY-algebra). To do so, we retain the non-trivial transformation law $\delta_\tau c = i_\tau A$ and we consider $\delta_\tau A$ to be unknown.

Let us evaluate the expression $\delta_\tau s A$ in terms of $\delta_\tau A$: by substituting the known expressions of sA and $\delta_\tau c$, and by using $d i_\tau = \mathcal{L}_\tau - d i_\tau$ as well as $dA + \frac{1}{2}[A, A] = F$, we obtain

$$\delta_\tau s A = \mathcal{L}_\tau A - i_\tau F + [c, \delta_\tau A]\,. \tag{4.21}$$

By virtue of the complete equation of motion for A, as given by equation (3.18), the classical contact term $i_\tau F$ in equation (4.21) can be expressed in terms of the contact term $i_\tau(\delta\Sigma^{CS}/\delta A)$. (In this way, the anti-ghosts enter our geometric framework which only involves classical and ghost fields.) Subsequent use of $b = s\bar{c}$ then entails

$$\delta_\tau s A = \mathcal{L}_\tau A - i_\tau \frac{\delta\Sigma^{CS}}{\delta A} - i_\tau(*ds\bar{c}) + [c, i_\tau(*d\bar{c})] + [c, \delta_\tau A]\,.$$

By adding the unknown quantity $s\delta_\tau A$ to both sides of this equation, we get the result

$$[s, \delta_\tau] A = \mathcal{L}_\tau A - i_\tau \frac{\delta\Sigma^{CS}}{\delta A} + s\{\delta_\tau A + i_\tau(*d\bar{c})\} + [c, \delta_\tau A + i_\tau(*d\bar{c})]\,. \tag{4.22}$$

Obviously, the choice

$$\delta_\tau A = -i_\tau(*d\bar{c}) \tag{4.23}$$

ensures the validity of the SUSY-algebra (3.5) and gives the known results (3.19) and (3.20). The requirement of invariance of the Chern-Simons action under the determined δ_τ-variations fixes the transformation laws of $\bar{c}$ and b, again in agreement with equations (3.19). (We could also argue that $\delta_\tau\bar{c}$ has to vanish for dimensional reasons; then $\delta_\tau b$ follows again by imposing the SUSY-algebra on $\bar{c} = sb$.)

Let us summarize once more our procedure: by starting from the $\emptyset$-type symmetry conditions, we could derive the VSUSY-transformations for the Chern-Simons theory solely from the knowledge of the total action and BRST-transformations and by assuming that the SUSY-algebra is fulfilled up to contact terms.

As we have shown in the appendix, the δ_τ-transformations of A and c can also be obtained in a direct way by redoing our initial derivation (4.3)–(4.5) after having determined the contact terms in the SUSY-algebra by dimensional arguments.

BF model. One proceeds as for the Chern-Simons theory. If we only consider the classical action, the $\emptyset$-type symmetry conditions (4.5) and (4.9) lead to equations (4.18) and to the following variations of the B-fields:

$$\begin{aligned}
\delta_\tau B &= 0 \\
\delta_\tau B^k_{p-k} &= i_\tau B^{k-1}_{p-k+1} \qquad \text{for } k = 1, \dots, p\,.
\end{aligned} \tag{4.24}$$

When extending these results to the complete gauge-fixed action, one has to take into account the fact that the SUSY-algebra is only valid on-shell and that it involves the complete equations of motion. By modifying the transformation laws $\delta_\tau A = 0$ and $\delta_\tau B = 0$ along the lines indicated above, one obtains the VSUSY-transformations (3.21) which fulfill the on-shell algebra (3.22).

5. Conclusion

From the previous considerations, we conclude that the VSUSY-transformations for Witten-type models follow straightforwardly from the 0-type or $\emptyset$-type symmetry conditions (their presence depending on which gauge-fixing condition is chosen). This derivation seems to be quite efficient, in particular for higher-dimensional TQFT's [19].

The VSUSY-transformations for Schwarz-type models follow from $\delta_\tau\tilde{A} = i_\tau\tilde{A}$ by checking the algebra and by taking into account the equations of motion of the model under consideration. An off-shell formulation for these theories can be obtained by considering the linearized Slavnov-Taylor operator which involves external sources. These sources are associated with the non-linear terms in the BRST-transformations and they also transform under the VSUSY which is now linearly broken [9]. Since

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the Batalin-Vilkovisky formalism naturally incorporates sources under the disguise of anti-fields (e.g. see references [22] for the application to topological models), it should represent a more convenient framework for discussing Schwarz-type models. This will be reported upon elsewhere [23].

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A. Another derivation of VSUSY-variations in Chern-Simons theory

In the following, we present a slightly different derivation of the VSUSY-transformations for Schwarz type models by using the Chern-Simons theory as an example. This approach is motivated by the results (4.18) and (4.19) which entail the vanishing of F, i.e. the classical, rather than the complete equation of motion for Chern-Simons theory. This fact exhibits the inadequacy of our starting point, i.e. of the assumption that the SUSY-algebra is fulfilled off-shell. Hence, we simply review the derivation (4.3)–(4.5) after having determined the contact terms in the SUSY-algebra (3.5) for the commutators $[s, \delta_\tau]A$ and $[s, \delta_\tau]c$. These terms can be found without explicitly knowing the δ_τ-variations since their form is strongly constrained: for the commutator $[s, \delta_\tau]A$, this term has to be a function of the functional derivatives of Σ^{CS} with respect to the fields of the model, and this function must be linear in τ and of the same dimension and ghost-number as A (similarly for the contact term in the commutator $[s, \delta_\tau]c$). From these arguments, we can deduce that the algebra can only have the form

$$\begin{aligned}
[s, \delta_\tau]A &= \mathcal{L}_\tau A + \xi\, i_\tau \frac{\delta \Sigma^{CS}}{\delta A} \\
[s, \delta_\tau]c &= \mathcal{L}_\tau c\,,
\end{aligned}$$

where ξ is a real factor. We now collect these two equations into a single one involving $\tilde{A} = A + c$ and we substitute the known expression (3.18) for the functional derivative:

$$[s, \delta_\tau]\tilde{A} = \mathcal{L}_\tau \tilde{A} + \xi\, i_\tau \{F + s(*d\bar{c}) - [c, *d\bar{c}]\}\,. \tag{A.1}$$

This relation represents the correct form of the SUSY-algebra for the present model. Thus, we substitute it in the δ_τ-variation (4.3) of $\tilde{F}$:

$$\delta_\tau \tilde{F} = \mathcal{L}_\tau \tilde{A} + \xi\, i_\tau F + \xi\, \{i_\tau s(*d\bar{c}) - [c, i_\tau(*d\bar{c})]\} - \tilde{D}(\delta_\tau \tilde{A})\,.$$

Next, we substitute the horizontality condition $\tilde{F} = F$ in $i_\tau F$ and eliminate $\mathcal{L}_\tau \tilde{A}$ by means of the general relation (4.1): if $\xi = -1$, the $i_\tau F$-term drops out from the last equation and we are left with

$$\delta_\tau \tilde{F} = \tilde{D}(i_\tau \tilde{A}) + s i_\tau(*d\bar{c}) - [c, i_\tau(*d\bar{c})] - \tilde{D}(\delta_\tau \tilde{A}) .$$

Thanks to $\tilde{D} \equiv \tilde{d} + [\tilde{A}, \cdot\,] = D + s + [c, \cdot\,]$, this result can be rewritten as

$$\delta_\tau \tilde{F} = -\tilde{D} \left[\delta_\tau \tilde{A} - i_\tau \tilde{A} + i_\tau(*d\bar{c}) \right] + D i_\tau(*d\bar{c}) . \tag{A.2}$$

Henceforth, the postulated conditions (4.6), which did not take into account the equations of motion, should be modified according to

$$\begin{aligned} \delta_\tau \tilde{A} &= i_\tau \tilde{A} - i_\tau(*d\bar{c}) \\ \delta_\tau \tilde{F} &= D i_\tau(*d\bar{c}) . \end{aligned} \tag{A.3}$$

Expansion with respect to the ghost-number now yields the known results (3.19).

In summary, the key point of this model-dependent derivation was to determine the general form of contact terms in the SUSY-algebra for the considered model.

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